Exercise sheet 11
Lecture p-adic Representation Theory

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On this problem sheet F denotes a non-archimedean local field. We write G for $\mathrm{GL}_2(F)$ and A for its Lie algebra.

Aufgabe 1. (10 points) Let $\psi : F \to \mathbb{C}^\times$ be a non-trivial smooth additive character of F and a be an element of $F^\times$. Recall that we denote by μ_ψ and μ_ψ^A the self-dual Haar measure relative to ψ on F and A, respectively. Prove the following equalities:

1. $\mu_{a\psi} = ||a||^{\frac{1}{2}}\mu_\psi$
2. $\mu_{a\psi}^A = ||a||^2\mu_\psi^A$.

Aufgabe 2. (10 points) Let χ be a smooth character of $F^\times$, ψ be a non-trivial smooth character of F, and we consider the map $\lambda_1 : C_c^\infty(F) \to \mathbb{C}(X)$ defined via:

$$\lambda_1(\Phi) := Z(\hat{\Phi}, \tilde{\chi}, \frac{1}{qX}),$$

where we take the Fourier transform with respect to μ_ψ. Prove that λ_1 satisfies:

$$\lambda_1(a\Phi) = \chi(a)X^{\nu_F(a)}\lambda_1(\Phi),$$

for all test functions $\Phi \in C_c^\infty(F)$ and $a \in F^\times$.

Aufgabe 3. (10 points) Let $\mathfrak{a}$ be an hereditary order of A. Let θ be an irreducible smooth representation of $U_\mathfrak{a}$ contained in an irreducible cuspidal representation (π, V) of $\mathrm{GL}_2(F)$. Consider the matrix coefficient f_0 of π for a pair $(\tilde{v}, v) \in \tilde{V}^{\tilde{\theta}} \times V^\theta$. Show the following identity for all $\Phi \in C_c^\infty(A)$:

$$Z(\Phi, f_0, X) = Z(e_\theta * \Phi * e_\theta, f_0, X).$$

Aufgabe 4. (10 Punkte) Calculate the local constant for an unramified character of $F^\times$ using the corresponding definitions.