Exercise sheet 10
Lecture p-adic Representation Theory

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On this problem sheet F denotes a non-archimedean local field. We denote by G_F the Galois group $\mathrm{Gal}(F^{sep}|F)$, and we use the usual notation for the Weil group $\mathcal{W}_F$ the inertia group $\mathcal{I}_F$ and the wild inertia group $\mathcal{P}_F$. All mentioned representations are complex representations.

Aufgabe 1. (10 points) Let (τ, W) be a smooth representation of the Weil group and suppose that the image of τ is a group of finite order f. Let $\alpha \mapsto \bar{\alpha}$ be the canonical projection from $\hat{\mathbb{Z}}$ to $\hat{\mathbb{Z}}/f\hat{\mathbb{Z}} \cong \mathbb{Z}/f\mathbb{Z}$. show that there is a Frobenius element Φ of G_F. such that the map $\rho: \ G_F \to \mathrm{Aut}_{\mathbb{C}}(W)$ defined via

$$\rho(g\Phi^{\alpha}) := \tau(g)\tau(\Phi)^{\bar{\alpha}}, \ g \in \mathcal{I}_F, \ \alpha \in \hat{\mathbb{Z}}$$

is a smooth representation of G_F.

Aufgabe 2. (10 points) Suppose that π is a representation of a group H and the latter be a finite index subgroup of a group G. Show that $\mathrm{Ind}_H^G \pi$ is semisimple if and only if π is semisimple.

Aufgabe 3. (10 points)(GL(2), Bushnell, Henniart, 31.2) A semisimple Deligne representation is called indecomposable if it cannot be written as a direct sum of Deligne representations. Show that every indecomposable Deligne representation is isomorphic to a representation of the form $\rho \otimes \mathrm{Sp}(n)$ where ρ is a smooth irreducible representation of the Weil group and $\mathrm{Sp}(n)$ is the n-dimensional special Deligne representation.