Exercise sheet 9
Lecture p-adic Representation Theory

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On this problem sheet F denotes a non-archimedean local field. We denote $G := \mathrm{GL}_2(F)$.

Aufgabe 1. (10 points) Show that for every cuspidal irreducible complex representation π of G there is a cuspidal type $(\mathfrak{a}, J, \Lambda)$, such that π is isomorphic to $\mathrm{Ind}_J^G \Lambda$.

Aufgabe 2. (10 points) Let π be an irreducible smooth complex representation. Show that π contains an essentielly scalar stratum if and only if there is a smooth character χ of $F^\times$ such that $l(\chi\pi) < l(\pi)$.

Aufgabe 3. (10 points) Let $(\mathfrak{a}, \Lambda, J)$ be a cuspidal type which contains a simple stratum $(\mathfrak{a}, n, \alpha)$. Suppose that n is odd. Show that Λ is one-dimensional.

Hint: Put $E := F[\alpha]$. The group J_α is by definition $E^\times U_{\mathfrak{a}}^{\lfloor \frac{n+1}{2} \rfloor}$ which is equal to $E^\times U_{\mathfrak{a}}^{\lfloor \frac{n}{2} \rfloor + 1}$, because n is odd. Thus we only need to consider ψ_α, which is defined on $U_{\mathfrak{a}}^{\lfloor \frac{n}{2} \rfloor + 1}$, and a character on $E^\times$.

Remark: If n is even, then J_α is bigger than $E^\times U_{\mathfrak{a}}^{\lfloor \frac{n}{2} \rfloor + 1}$, because $\lfloor \frac{n}{2} \rfloor + 1$ is greater than $\lfloor \frac{n+1}{2} \rfloor$, and Λ is not a character anymore.

Aufgabe 4. (10 points) Let $\mathcal{W}_F$ be the Weil group of F, and let σ be a smooth finite dimensional representation of $\mathcal{W}_F$. Show that for every Frobenius element Ψ of $\mathcal{W}_F$ there is an integer a such that $\sigma(\Psi^a)$ commutes with all elements of $\sigma(\mathcal{I}_F)$ where $\mathcal{I}_F$ is the inertia group of F.