Exercise sheet 7
Lecture p-adic Representation Theory

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On this problem sheet F denotes a non-archimedean local field.

Aufgabe 1. (5 points) Let $0 \to W_1 \to W_2 \to W_3 \to 0$ be a short exact sequence of smooth representations of an abelian locally profinite group G, s.t. W_1 and W_3 are one-dimensional and non-isomorphic. Show that the sequence splits.

Aufgabe 2. (10 points)(Steinberg representation) Show for $G = \mathrm{GL}_2(F)$ the isomorphy $\mathrm{St}_G \cong \widetilde{St}_G$.

Aufgabe 3. (10 points) Let T be the standard torus of $G = \mathrm{GL}_2(F)$ and let χ and χ' be two smooth characters of T. Show that $\mathrm{Hom}_G(\mathrm{Ind}_B^G\chi, \mathrm{Ind}_B^G\chi')$ is non-zero if and only if $\chi = \chi'$ or $\chi' = {}^{\omega}\chi\delta_B$.

Aufgabe 4. (10 points) Let χ be a smooth character of the standard torus T of $G = \mathrm{GL}_2(F)$. We consider the standard Borel subgroup $B = TN$. Let Y be the B subrepresentation of $\mathrm{Ind}_B^G\chi$ consisting of those elements f of $\mathrm{Ind}_B^G\chi$ which satisfy $f(1) = 0$. Show that $\mathrm{Res}_N^B Y$ is isomorophic to $C_c^\infty(N)$, the latter given with the usual left action r: $(r(n).h)(n') := h(n'n),\ h \in C_c^\infty(N)$. Hint: The map was given in the lecture.