Exercise sheet 6
Lecture p-adic Representation Theory

Dr. Daniel Skodlerack

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In this problem sheet F denotes a non-archimedean local field.

Aufgabe 1. (10 points) Show that there is a non-semisimple two-dimensional smooth complex representation of $\mathrm{GL}_n(F)$. Hint: Show at first that there is a non-semisimple two-dimensional complex representation of $\mathbb{Z}$.

Aufgabe 2. (10 points)(Mackey's irreducibility criteria) Let G be a second countable locally profinite group and H be an open subgroup of G, such that for all $g \in G$ the space $(H \cap gHg^{-1})\backslash H$ is finite. (For example the condition on H is satisfied if H is open contains the center Z of G and is compact mod Z.) Then the following assertions are equivalent for a smooth irreducible representation (σ, W) of H.

1. $\text{c-Ind}_H^G \sigma$ is irreducible.
2. The intertwining of σ in G is equal to H.
3. $\mathrm{End}_G(\text{c-Ind}_H^G \sigma) \cong \mathbb{C}$.

Aufgabe 3. (10 points)(Producing cuspidal representations) Suppose that (σ, W) is an irreducible representation of an open subgroup H of $\mathrm{GL}_n(F)$ which contains and is compact mod the center of $\mathrm{GL}_n(F)$. Suppose that the intertwining of σ in G is equal to H. Then $\text{c-Ind}_H^G \sigma$ is irreducible and cuspidal.

Aufgabe 4. (5 points) Find the cuspidal support of the Steinberg representation of $\mathrm{GL}_2(F)$, i.e. of the infinite dimensional irreducible quotient of $\mathrm{Ind}_B^{\mathrm{GL}_2(F)} \mathbb{1}$, where B is the standard (upper) Borel subgroup.