Exercise sheet 4
Lecture p-adic Representation Theory

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Aufgabe 1. (5+5 points)

1. Every smooth irreducible complex representation of a second countable abelian locally profinite group is one dimensional.
2. Does the statement of 1 still remains true if we skip the second countability?

Aufgabe 2. (5+5+5* points) Let $\mathbb{1}$ be the trivial representation on $o_F^\times$.

1. Show that $(\pi, V) := \text{c-Ind}_{o_F^\times}^{F^\times} \mathbb{1}$ is isomorphic to $\mathbb{C}[X, X^{-1}]$ as a $\mathbb{C}$-vector space, say via ϕ. Then the $F^\times$-action on V and ϕ define an $F^\times$-action on $\mathbb{C}[X, X^{-1}]$ via pushforward. Calculate this action in terms of $\mathbb{C}[X, X^{-1}]$ using your choice of ϕ.
2. Show that (π, V) is not irreducible, not semisimple, not admissible and that it has an irreducible quotient, but no irreducible subrepresentation.
3. Show that $\text{Ind}_{o_F^\times}^{F^\times} \mathbb{1}$ has no irreducible quotient.

Aufgabe 3. (5 points) Let (π, V) be an admissible smooth complex representation. Show that V is irreducible if and only if its contragredient is irreducible.

Aufgabe 4. (5 points) Let G be a locally profinite group and χ be a smooth character of G. Let further H be a closed subgroup of G and (ρ, W) be a complex smooth representation of H. Prove $\text{c-Ind}_H^G(\rho\chi|_H) \cong (\text{c-Ind}_H^G(\rho))\chi$ and $\text{Ind}_H^G(\rho\chi|_H) \cong (\text{Ind}_H^G(\rho))\chi$.

Aufgabe 5. (5+5 points) Let G be a locally profinite group and $H_1 \leq H_2$ closed subgroups of G. Show the transitivity of induction and compact induction, i.e.

$$\text{c-Ind}_{H_2}^G \text{c-Ind}_{H_1}^{H_2} \rho \cong \text{c-Ind}_{H_1}^G \rho$$

and

$$\text{Ind}_{H_2}^G \text{Ind}_{H_1}^{H_2} \rho \cong \text{Ind}_{H_1}^G \rho$$

for all smooth representations ρ of H.