Exercise sheet 3
Lecture p-adic Representation Theory

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Aufgabe 1 (5+5 points).
1. Let (π, V) be an irreducible smooth complex representation of a profinite group. Show that V is finite dimensional.
2. Show that every smooth complex representation of a profinite group K is semisimple. Hint: Show that every smooth complex representation of K is unitary and restrict to the finite dimensional case.

Aufgabe 2 (5 points). Find a smooth irreducible complex representation of $\mathrm{GL}_2(\mathbb{Z}_2)$ which is not a character, i.e. which is not one-dimensional.

Aufgabe 3 (5 points). Let F be a non-archimedian local field. Then every smooth complex character of $\mathrm{GL}_m(F)$ has the form $\chi \circ \det$ where χ is a smooth complex character of $F^\times$.

Aufgabe 4 (5+5+5* points). We consider a smooth irreducible representation ρ of a profinite group K, and we define $e_\rho \in C_c^\infty(K)$ via

$$e_\rho(g) := \frac{\dim(\rho)}{\mu(K)} \mathrm{tr}(\rho(g^{-1})).$$

1. Show that e_ρ is an idempotent in $\mathcal{H}(K)$, using the convolution formular for traces for representations of finite groups.
2. Show that e_ρ is central in $\mathcal{H}(K)$.
3. Let (π, V) be a smooth representation of K. Show that $\pi(e_\rho)V$ is the ρ-isotypic component of V, i.e. $\pi(e_\rho)V$ is the sum of all subrepresentations of V which are isomorphic to ρ.