Exercise sheet 2
Lecture p-adic Representation Theory

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Aufgabe 1 (5 points)**.** Let G be a totally disconnected locally compact group (t.d.l.c. group). Prove that G has a neighbourhood base of e consisting of open compact subgroups. Hint: Consider $S = \{u \in U | uU \subseteq U\}$ for a clopen set U.

Aufgabe 2 (3+3+5 points)**.** Let G be a t.d.l.c. group and let K be a normal compact open subgroup of G and K' be a normal open subgroup of K. We fix a Haar measure μ on G.

1. Let g be an element of G. The dirac distribution δ_g of g is defined to be the $\mathbb{C}$-linear form on $C^\infty(G)$ given by $\delta_g(f) := f(g)$. Which element of $C_c^\infty(G)$ corresponds to $e_{K'} * \delta_g * e_{K'}$ under the isomorphism $C_c^\infty(G) \cong \mathcal{H}(G)$ which was defined as $f \mapsto f d\mu$.

2. Calculate the Hecke algebra $\mathcal{H}(G//K)$.

3. Let $\{k_i | i \in I\}$ be system of representatives of K/K', and let S be the set of those elements of $C_c^\infty(K' \backslash G/K')$ which satisfy $\sum_i f(gk_i) = 0$ for all elements $g \in G$. Prove that $\mathcal{H}(G//K')/S$ is isomorphic to $\mathcal{H}(G/K)$.

Aufgabe 3 (5 points)**.** Calculate the modulus character of a maximal parabolic subgroup of $\mathrm{GL}_3(F)$, where F is a non-archimedian local field.

Aufgabe 4 (5 points)**.** Let n be a natural number. Show that $\mathrm{GL}_n(F)$ is unimodular. Hint: Iwasawa decomposition.