# ADVANCED ALGEBRA, FALL 2026
# PROBLEM SHEET 6

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**Problem 1** (10, closure in the k-topology)**.** We fix $L|k$ and $m$ as in Chapter IV of the lecture. Let $Y$ be a subset of $L^m$. Let $V$ be the closure of $Y$ in $L^m$ in the $k$-topology. Find the associated ideal of $V$ in $k[X_1, \dots, X_m]$.

**Problem 2** (10, dimension with respect to an algebraic extension))**.** Let $L$ be an algebraic closure of a field $k$ and $m$ a positive integer. Further let $V$ be a $k$-algebraic set of $L^m$ Show that

$$k - \dim V = L - \dim V.$$

**Problem 3** (10+10, generic point)**.** Let $L$ be an algebraic closure of a field $k$ and $m$ a positive integer. Further let $V$ be a $k$-algebraic set of $L^m$ and suppose that $V$ contains a generic point.

(i) Prove that $V$ is finite.
(ii) For $n \in \mathbb{N}$, find an example for $|V| = n$.

*Date*: Please hand in your solution before the lecture on Tuesday, **November 10th 2026**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.