

HOMOLOGICAL ALGEBRA, FALL 2025
PROBLEM SHEET 14

PROF. DANIEL SKODLERACK

Problem 1 (20, Gysin sequence). Prove Theorem 191..

Problem 2 (10, completion). Prove that the completion of a filtered chain complex C defines a spectral sequence which is isomorphic to the spectral sequence of C .

Problem 3 (10, exhaustion and Hausdorff). Let (C, F) be filtered chain complex. Show that $C, \cup_p F_p C$, and $C / \cap_p F_p C$ define the same spectral sequence.

Problem 4 (10, concrete example). Consider the chain complex C :

$$0 \rightarrow \mathbb{Z} \xrightarrow{12} \mathbb{Z} \xrightarrow{3} \mathbb{Z}/12 \rightarrow 0.$$

with filtration $F_p C := 3^{-p} C$ for $p \in \mathbb{Z}^{\leq 0}$. Find the spectral sequence of (C, F) .

Date: Please hand in before the lecture on Friday, **December 26thth 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.