

**HOMOLOGICAL ALGEBRA, FALL 2025**  
**PROBLEM SHEET 14**

PROF. DANIEL SKODLERACK

**Problem 1** (20, Gysin sequence). Prove Theorem 19.1.

**Problem 2** (10, completion). Prove that the completion of a filtered chain complex  $C$  defines a spectral sequence which is isomorphic to the spectral sequence of  $C$ .

**Problem 3** (10, exhaustion and Hausdorff). Let  $(C, F)$  be a filtered chain complex. Show that  $C, \cup_p F_p C$ , and  $C / \cap_p F_p C$  define the same spectral sequence.

**Problem 4** (10, concrete example). Consider the chain complex  $C$ :

$$0 \rightarrow \mathbb{Z} \xrightarrow{12} \mathbb{Z} \xrightarrow{3} \mathbb{Z}/12 \rightarrow 0.$$

with filtration  $F_p C := 3^{-p} C$  for  $p \in \mathbb{Z}^{\leq 0}$ . Find the spectral sequence of  $(C, F)$ .

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*Date:* Please hand in before the lecture on Friday, **December 26th<sup>th</sup> 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.