HOMOLOGICAL ALGEBRA, FALL 2025
PROBLEM SHEET 15

PROF. DANIEL SKODLERACK

Problem 1 (20*, hyper derived functors as usual derived functors on $Ch_{\geqslant 0}$)**.** Solve Exercise 205(i), see [Wei94, Exercise 5.7.4 (2.)].

Problem 2 (10, lifting chain morphisms to Cartan-Eilenberg resolutions)**.** Let $P \to A$ and $Q \to B$ be Cartan-Eilenberg resolutions and $f : A \to B$ be a chain morphism. Prove that there is a morphism of double complexes $\tilde{f} : P \to Q$ lying over f. (See Exercise 201)

Problem 3 (20, homotopic lifts to Cartan-Eilenberg resolutions)**.** Under the assumptions of Problem 2 suppose we are given a second chain morphism $g : A \to B$ chain homotopic to f. Prove that $\tilde{g} : P \to Q$ is chain homotopic to $\tilde{f}$.

Problem 4 (10, exact triangles)**.** Solve [Wei94, Exercise 10.1.2]. It gives an example of a short exact sequence of abelian groups which cannot be made to an exact triangle in the homotopy category of abelian groups.

REFERENCES

[Wei94] Charles A. Weibel. *An introduction to homological algebra*, volume 38 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1994.