HOMOLOGICAL ALGEBRA, FALL 2025
PROBLEM SHEET 13

PROF. DANIEL SKODLERACK

Problem 1 (20, mapping lemma)**.** Prove the mapping lemma, see Lemma 174.

Problem 2 (10, loop space)**.** Compute all homology groups for the based loop space of S^n, $n \geqslant 2$, see Exercise 189.

Problem 3 (10, complex projective space)**.** Compute all homology groups of the projective space $\mathbb{C}P^n$, $n \geqslant 1$, see [Wei94, 5.3.2]

Problem 4 (10*, double complex)**.** Is there a double complex with a homology spectral sequence which doesn't converge to the homology of the direct sum total complex?

References

[Wei94] Charles A. Weibel. *An introduction to homological algebra*, volume 38 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1994.

Date: Please hand in before the lecture on Friday, **December 19th$^{\text{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.