HOMOLOGICAL ALGEBRA, FALL 2025
PROBLEM SHEET 11

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Problem 1 (20, right and left global dimension)**.** Consider the ring

$$R = \begin{pmatrix} \mathbb{Z} & \mathbb{Q} \\ 0 & \mathbb{Q} \end{pmatrix}$$

(i) Prove that the ring R is right noetherian.
(ii) Prove that R is not left noetherian.
(iii) Show that the right global dimension of the ring R is 1.
(iv) Show that the left global dimension of the ring R is 2.

Problem 2 (10, $\mathbb{Z}[X]$)**.** Find the Tor-dimension of $\mathbb{Z}[X]$. *Hint: You can use that every ideal of* $\mathbb{Z}[X]$ *can be generated by two elements.*

Problem 3 (10+10*, matrix rings)**.** Find the Tor-dimension of $M_2(\mathbb{R})$ and $M_2(\mathbb{Z})$.

Date: Please hand in before the lecture on Friday, **December 5th$^{\text{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.