HOMOLOGICAL ALGEBRA, FALL 2025
PROBLEM SHEET 10

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Problem 1 (10, Ext)**.** Conpute $\mathrm{Ext}^i_{\mathbb{Z}}(\mathbb{Q}^\times, \mathbb{Z})$ for $i = 0, 1$.

Problem 2 (15, Ext and extensions)**.** Solve the three exercises stated in Theorem 135 of the lecture notes.

Problem 3 (10, extensions)**.** Let p be a prime number. Find all classes of extensions of $\mathbb{Z}/p$ by $\mathbb{Z}/p$ in the category of abelian groups. You need to prove that your findings are pairwise not equivalent.

Problem 4 (10, injective modules)**.** Prove that an R-module N is injective if and only if for all R-modules M we have $\mathrm{Ext}^1_R(M, N) = 0$.

Date: Please hand in before the lecture on Friday, **November 28th$^{\text{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.