HOMOLOGICAL ALGEBRA, FALL 2025
PROBLEM SHEET 9

PROF. DANIEL SKODLERACK

Problem 1 (10, direct limit)**.** Solve Exercise 114 in the lecture notes.

Problem 2 (10, Pontrjagin dual)**.** Let $0 \to A \to B \to C \to 0$ be a short complex of R-left modules. Prove that this sequence is axact if and only if $0 \to A^* \to B^* \to C^* \to 0$ is exact.

Problem 3 (10+10, double dual)**.** There is a canonical map for R-modules $A \to (A^*)^*$. Prove that this map is injective. Is it always surjective?

Problem 4 (10, torsion-free)**.** Prove or disprove that a flat module over an integral domain has to be torsion-free.

Date: Please hand in before the lecture on Friday, **November 21stth 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.