HOMOLOGICAL ALGEBRA, FALL 2025
PROBLEM SHEET 8

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Note that if not emphasised differently, all functors are meant to be covariant.

Problem 1 (10, double complex)**.** Compute the homology groups of $Tot^{\prod}(C)$ where $C_{p,q} = \mathbb{Z}/4$ for all integers p, q and all differentials are the multiplication by 2. Show that $Tot^{\oplus}(C)$ is acyclic.

Problem 2 (10+10**, left exact vis-a-vis right exact)**.** Let $F : \mathcal{A} \to \mathcal{B}$ be a left exact covariant functor and suppose that $\mathcal{A}$ has enough projectives. Prove that for all non-negative integers i we have:

$$R^i F(A) \cong (L_i(F^{op}))^{op}(A).$$

for all $A \in \mathcal{A}$,

(i) for module categories.
(ii) for general abelian categories.

Problem 3 (10, Ext)**.** Prove Proposition 94 without using that $Ext^i(A, B)$ can be computed via resolving the first entry A.

Problem 4 (10, sheaf cohomology)**.** Prove that the global sections functor in Example 97(b) is left exact.

Date: Please hand in before the lecture on Friday, **November 14th$^{\text{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.