

HOMOLOGICAL ALGEBRA, FALL 2025
PROBLEM SHEET 6

PROF. DANIEL SKODLERACK

Problem 1 (10, [Injectives in the category of sheaves of abelian groups](#)). Solve [Wei94, Exercise 2.3.6].

Problem 2 (20, [adjoint pair](#)). Let (L, R) be an adjoint pair of additive covariant functors between abelian categories. Prove:

- (i) R is left exact.
- (ii) L is right exact.

Let L' be another additive left adjoint of R . Prove that there is a natural isomorphism from L to L' .

Problem 3 (20, [covariant Hom functor](#)). Let M be a finitely generated right R -module. Is there

- (i) a covariant additive right adjoint of $\text{Hom}_R(M, -)$?
- (ii) a covariant additive left adjoint of $\text{Hom}_R(M, -)$?

Here we consider $\text{Hom}(M, -)$ as a functor from $R\text{-mod}$ to Ab .

REFERENCES

[Wei94] Charles A. Weibel. *An introduction to homological algebra*, volume 38 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1994.

Date: Please hand in before the lecture on Friday, **October 31stth 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.