# HOMOLOGICAL ALGEBRA, FALL 2025
# PROBLEM SHEET 4

PROF. DANIEL SKODLERACK

**Problem 1** (10, mapping cone)**.** Let $f$ be a chain map between chain complexes of $R$-modules. Suppose that $f$ is a quasi-isomorphism. Prove that $H_{n-2}(ker(f)) \oplus H_{n-1}(im(f))$ is $R$-isomorphic to $H_n(coker(f)) \oplus H_{n-1}(im(f))$, for all integers $n$.

**Problem 2** (10, homotopy equivalence for the cone)**.** Prove Exercise 1.5.5 in [Wei94].

**Problem 3** (40, first property of the mapping cylinder)**.** Prove Proposition 46.

**Problem 4** (10, cone in an exact triangle)**.** Prove for a chain map $f : B \to C$ in $Ch(\mathcal{A})$ for a module category $\mathcal{A}$ that the composition of $f$ with the canonical chain map

$$pr_1 : cone(f)[1] \to B,$$

i.e. $f \circ pr_1$, is null-homotopic.

## References

[Wei94] Charles A. Weibel. *An introduction to homological algebra*, volume 38 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1994.

*Date*: Please hand in before the lecture on Friday, **October 17th$^{\mathbf{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.