HOMOLOGICAL ALGEBRA, FALL 2025
PROBLEM SHEET 2

PROF. DANIEL SKODLERACK

Problem 1 (10, acyclic total complex)**.** Suppose C is a bounded double complex of abelian groups. Suppose that all rows are exact. Prove that the total complex associated to C is exact.

Problem 2 (10, non-acyclic total complexes)**.** Find a double complex of R-modules where all rows and columns are exact, but both associated total complexes are not acyclic.

Problem 3 (20, homology group of a total complex)**.** Consider Example 16(a) but for the complexes of abelian groups

$$A : 0 \to \mathbb{Z} \xrightarrow{2\iota_1} \mathbb{Z} \oplus \mathbb{Z} \xrightarrow{3pr_2} \mathbb{Z} \to 0$$

with amplitude in $[-3, -1]$ and

$$B : 0 \to \mathbb{Z} \xrightarrow{8} \mathbb{Z} \xrightarrow{\pi} \mathbb{Z}/2\mathbb{Z} \to 0$$

with amplitude $[1, 3]$. Compute the homology groups of the total complex associated to the double complex C.

Date: Please hand in before the lecture on Friday, **September 26th 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.