HOMOLOGICAL ALGEBRA, FALL 2025
PROBLEM SHEET 1

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Problem 1 (10+10*, categories)**.** (i) Is there a category which is Ab but not additive? Prove your statement.

(ii) (*) Give an example of an additive category which is not abelian.

Problem 2 (10*, axioms for abelian categories)**.** Let $\mathcal{A}$ be an additive category and suppose it satisfies Axiom (5) and (6) of Definition 11 in the notes. Does follow that Axiom (7) is satisfied for $\mathcal{A}$? Prove your answer.

Problem 3 (10, category of left R-modules)**.** Let $f : A \to B$ be an R-module homomorphism. Prove that the following statements are equivalent:

(i) f is a monic.
(ii) f is injective.
(iii) f is a kernel of a map (in the categorical sense).

Problem 4 (10, homology of a graph)**.** Consider the graph in Example 9. Prove that the first homology group is free of rank 2, without using the general formula for the first homology group of a graph.

Problem 5 (10, homology of a surface of a tetrahedron)**.** Compute the simplicial homology groups with coefficients in R for the surface of a tetrahedron. Prove your results.

Date: Please hand in before the lecture on Friday, **September 19th 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.