DIFFERENTIAL TOPOLOGY 1 SPRING 2025
PROBLEM SHEET 12

PROF. DANIEL SKODLERACK

Problem 1 (10, punctured manifold)**.** (i) Let M be a connected manifold without boundary of positive dimension and $P \in M$. Compute all compact cohomology groups of $M \backslash \{P\}$.

(ii) Compute the first compact cohomology group of a punctured cylinder without boundary, i.e. of $(S^1 \times \mathbb{R}) \backslash \{(1, 0, 0)\}$.

Problem 2 (10, de Rham cohomology and compact de Rham cohomology)**.** Compute the first de Rham cohomology group and the first compact de Rham cohomology group of $\mathbb{R}^2 \backslash (\mathbb{Z} \times \{0\}$.

Problem 3 (10, non-degenerate bilinear form)**.** Let V and W be finite dimensional vector spaces and $b : V \times W \to \mathbb{R}$ be a bilinear form. Prove that the following assertions are equivalent.

(i) b is non-degenerate, i.e. for all non-zero $v \in V$ there is a $w \in W$ such that $b(v, w) \neq 0$ and for all non-zero $w \in W$ there is a $v \in V$ such that $b(v, w) \neq 0$.

(ii) The map $v \in V \mapsto b(v, -) \in W^*$ is an isomorphism.

Date: Please hand in before the lecture on Thursday, **May 15th$^{\mathbf{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.