DIFFERENTIAL TOPOLOGY 1 SPRING 2025
PROBLEM SHEET 9

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Problem 1 (10, orientation)**.** Prove that a connected manifold cannot have exactly one orientation.

Problem 2 (10, nowhere vanishing top-degree form)**.** Prove that 2 implies 3 in Theorem 148, i.e. that an m-dimensional manifold with an atlas where all transition maps have positive determinant must admit a nowhere vanishing degree m differential form.

Problem 3 (10, orientation induced on the boundary)**.** In Proposition 150 we have defined the map $\partial\mathfrak{o}$ on ∂M. Prove that $\partial\mathfrak{o}$ is an orientation on ∂M.

Problem 4 (10, some integral)**.** Let a, b, c be positive real numbers and M be the following manifold

$$M = \left\{ (x, y, z) \in \mathbb{R}^3 \,\middle|\, \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1 \right\},$$

and let $\mathfrak{o}$ be the orientation on M such that $dx \wedge dy$ has orientation $\mathfrak{o}$ at $P = (0, 0, -c)$. Compute

$$\int_{M,\mathfrak{o}} (\cosh(z^2) + z) dx \wedge dy.$$

Date: Please hand in before the lecture on Thursday, **April 24$^{\text{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.