DIFFERENTIAL TOPOLOGY 1 SPRING 2025
PROBLEM SHEET 8

PROF. DANIEL SKODLERACK

Problem 1 (10, de Rham cohomology of the projective plane)**.** Prove that the map $\beta^{(1)}$ in the Mayer-Vietoris sequence for the projective plane, see Example 133 (c), has the form $\beta^{(1)}(x) = 2x,\ x \in \mathbb{R}$.

Problem 2 (10+10+10+10, Mayer-Vietoris sequence)**.** Compute the de Rham cohomology groups for

(i) the n-sphere, $n > 2$.
(ii) the n-dimensional projective space, $n > 2$.
(iii) the Kleinian bottle.
(iv) $S^1 \times S^1 \times \ldots \times S^1$, n copies of S^1.

Problem 3 (10**, shrinking)**.** Prove that every open cover of a manifold has a shrinking.

Date: Please hand in before the lecture on Thursday, **April 17$^{\text{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.