DIFFERENTIAL TOPOLOGY 1 SPRING 2025
PROBLEM SHEET 7

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Problem 1 (10, integral method)**.** Verify $df = \rho$ in Example 103(h). Hint: You can transfer to polar coordinates.

Problem 2 (10, Künneth formula)**.** Let $M,\ N$ be two manifolds. Consider the tensor product of the two de Rham complexes:

$$\tilde{\Omega}^k := \bigoplus_{i+j=k} \Omega^i(M) \otimes_{\mathbb{R}} \Omega^j(N).$$

(i) Prove that the canonical map $F : \tilde{\Omega}^* \to \Omega^*(M \times N)$ is injective.
(ii) Which differential does $d_{M\times N}$ induce on $\tilde{\Omega}^*$?
(iii) Is F surjective?

Problem 3 (10, pullback under projection)**.** Consider the projection $pr : \mathbb{R}^m \to \mathbb{R}^{m-1}$ which maps $(t_1, \ldots, t_m)$ to $(t_1, \ldots, t_{m-1})$ and a smooth differential k-form $\omega = \Sigma_{|I|=k} a_I dx_I$ on $\mathbb{R}^m$. Prove that the following statements are equivalent:

(i) ω is in the image of pr^*.
(ii) For all I the partial derivative of a_I with respect to x_m is identically zero and for all I containing m the map a_I is identically zero.

Problem 4 (10, pullback)**.** Consider the map $F : \mathbb{R}^2 \to \mathbb{R}^2,\ F(x, y) = (x^3 + y^2, xy)$. Let (u, v) be the standard coordinates on the codomain $\mathbb{R}^2$. Compute $F^*(udu + dv)$. (The idea of the problem has been taken from [**?**, Problem 19.2], but it is not the same problem.)

Problem 5 (10*, homotopy lemma)**.** Prove Case 1 in the proof of Lemma 125.

Date: Please hand in before the lecture on Thursday, **April 10th 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.