DIFFERENTIAL TOPOLOGY 1 SPRING 2025
PROBLEM SHEET 5

PROF. DANIEL SKODLERACK

Problem 1 (10, vector bundle structure by frames)**.** Read and prove Lemma 100 in the notes, see the Week 5 notes.

Problem 2 (10, wedge product)**.** Prove Lemma 89(b).

Problem 3 (10, pull back of a vector bundle)**.** Let $f : M \to N$ be a smooth map between manifolds and (F, N, ρ) be a vector bundle on N. Show that there exists a vector bundle (E, M, π) on M such that on fibres of f the bundle is trivial and such that there is a bundle map $\tilde{f} : E \to F$ over f which is an isomorphism on fibres.

Problem 4 (10, normal bundle over S^1)**.** We consider the normal line bundle $L := (TS^1)^\perp$ of S^1 as a sub-bundle of $T\mathbb{R}^2$. Is there a sub-bundle of $T\mathbb{R}^2$ of rank 1 extending L? You can use that any vector bundle over a contractible manifold is trivialisable, see [Hir76, Ch. 4 Cor. 2.5].

Problem 5 (10*, exterior product)**.** Complete the proof of Proposition 92, i.e. prove the injectivity of Ψ.

References

[Hir76] Morris W. Hirsch. *Differential topology*, volume No. 33 of *Graduate Texts in Mathematics*. Springer-Verlag, New York-Heidelberg, 1976.

Date: Please hand in before the lecture on Thursday, **March 27$^{\text{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.