DIFFERENTIAL TOPOLOGY 1 SPRING 2025
PROBLEM SHEET 4

PROF. DANIEL SKODLERACK

Problem 1 (10, boundary point of a chart)**.** Given a topological manifold M, a point P of M, and two topological charts (U, ϕ), (V, ψ) such that $P \in U \cap V$. Suppose that P is a boundary point of (U, ϕ). Prove that P is a boundary point of (V, ψ).

Problem 2 (10, embedding between manifolds with boundary)**.** Prove Proposition 56 for manifolds with boundary.

Problem 3 (10, boundary manifold)**.** Let M be a manifold with non-empty boundary. Prove that ∂M has the structure of a manifold.

Problem 4 (10, manifold minus an open ball)**.** Let M be a manifold without boundary and (U, ϕ) be a chart and B be an open ball in $\phi(U)$ such that the closed ball $\bar{B}$ is contained in $\phi(U)$. Prove that $M \backslash \phi^{-1}(B)$ is a manifold with boundary $\phi^{-1}(\partial \bar{B})$.

Problem 5 (10*, neat embedding)**.** Given any submanifold M of $\mathbb{R}^3$ is there a neat embedding of M into $\mathbb{R}^3$? If yes, give a proof. If not, then present a counterexample with proof that it is a counterexample.

Date: Please hand in before the lecture on Thursday, **March 20$^{\text{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.