# DIFFERENTIAL TOPOLOGY 1 SPRING 2025
# PROBLEM SHEET 3

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**Problem 1** (10, vector fields)**.** Prove Proposition 52.

**Problem 2** (10, real projective plane)**.**

(i) Prove that the map in Example 65 (a) is an embedding.

(ii) Prove that the tangent bundle of the projective plane is not trivializable.

**Problem 3** (10, critical values)**.** Find all critical values of the following map and draw their fibres.

$$f : \mathbb{R}^3 \to \mathbb{R}^2,\ F(x, y, z) = (x^2 - y^2, xyz)^T.$$

**Problem 4** (10, manifold structure on $TM$)**.** Let $(U, \phi = (x_1, x_2, \ldots, x_m))$ be a chart of a manifold $M$.

(i) Prove that the map form $U$ to $TM$ given by

$$P \mapsto \frac{\partial}{\partial x_1}(P)$$

is smooth.

(ii) Let $(V, \psi = (y_1, \ldots, y_m))$ be a second chart with $U = V$. Then for every $P \in U$ there is a matrix $A(P) \in \mathbb{R}^{m\times m}$ such that

$$(\frac{\partial}{\partial x_1}(P), \ldots, \frac{\partial}{\partial x_m}(P)) = (\frac{\partial}{\partial y_1}(P), \ldots, \frac{\partial}{\partial y_m}(P))A(P).$$

Compute $A(P)$.

**Problem 5** (10*, closed embedding of a line in a non-compact manifold)**.** Let $M$ be a connected manifold. Then are equivalent.

(i) $M$ is not compact.

(ii) There is a closed embedding of $\mathbb{R}$ into $M$. ("Closed" here means that the image is closed in $M$)

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*Date*: Please hand in before the lecture on Thursday, **March 13$^{\text{th}}$ 2025**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.