LINEAR ALGEBRA 1 (FALL 2024)
PROBLEM SHEET 13

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Problem 1 (20, Least square)**.** Find the least square quadratic fit $y = a_0 + a_1 x + a_2 x^2$ to the data points

$$(1,-2), (0,-1), (1,0), (2,4),$$

see Problem 4 on Page 393 in the textbook, Exercise set 6.5.

Problem 2 (20, QR-decomposition)**.** Find the QR-decomposition of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 2 & -1 & 1 & 1 \end{pmatrix}$$

Problem 3 (15, norm)**.** Let $(\mathrm{V}, \langle , \rangle)$ be an inner product space. Prove that $\| \, \| : \mathrm{V} \to \mathbb{R}$, given by $\|v\| := \sqrt{\langle v, v \rangle}$, is a norm on V. (See Theorem 123 in the notes for the definition of a norm.)

Problem 4 (10, linear maps)**.** Find a linear map $f : \mathbb{R}^3 \to \mathbb{R}^3$ which has $(1,1,1)^T$ in its kernel and both vectors $(1,1,-1)^T, (1,1,2)^T$ in its range.

Problem 5 (10+10*, projection)**.** Let A be a real $m \times n$ matrix of rank n and let W be its column space. Prove that the orthogonal projection onto W satisfies

$$\mathrm{proj}_{\mathrm{W}}(v) = \mathrm{A}(\mathrm{A}^T \mathrm{A})^{-1} \mathrm{A}^T v, \; v \in \mathbb{R}^m.$$

Find $\mathrm{proj}_{\mathrm{W}}(e_1)$ for the first two matrices in 12.4. (Hint: There are different ways to compute the projection map, you have already the QR-decomposition. Look at Theorem 281 in the notes)

Problem 6 (15*, orthogonal matrices)**.** Prove Proposition 284 (Page 337 in the notes)

Date: Please hand in before the lecture by **25th of December 2024**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference. * questions give extra points.