LINEAR ALGEBRA 1 (FALL 2024)
PROBLEM SHEET 8

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Problem 1 (10+10+10+5+5, rank of a matrix)**.** Consider the following matrix:

$$\mathrm{A} = \begin{pmatrix} 2 & 1 & 5 & 2 & 3 \\ 1 & 2 & 1 & 1 & 2 \\ 3 & 2 & 7 & 2 & 1 \\ 0 & 1 & -1 & 1 & 4 \end{pmatrix}.$$

Compute

(i) a basis for null(A).
(ii) a basis for col(A).
(iii) a basis for row(A).
(iv) nullity(A).
(v) the rank of A.

Problem 2 (10*+10+10, infinite dimension)**.** We also have the notion of linear independence for infinite sets: Let V be a vector space and T be a subset of V. We call T *linearly independent* if all finite subsets of T are linearly independent, see Definition 168 for finite sets. A subset B of V is called a *basis* of V if it spans V and is linearly independent.

(i) (*) Prove: V is finitely generated, i.e. there exists a finite set S which spans V, if and only if every linearly independent subset of V is finite.
(ii) Show that Poly($\mathbb{R}$) (the set of polynomial functions of $\mathbb{R}$) is not finitely generated.
(iii) Find a basis for

$$\{p \in \mathrm{Poly}(\mathbb{R}) \mid \ \forall x \in \mathbb{R} : \ p(-x) = p(x)\},$$

the space of even polynomial functions.

Date: Please hand in before the lecture by **15th of November 2024**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference. * questions give extra points.