LINEAR ALGEBRA 1 (FALL 2024)
PROBLEM SHEET 5

PROF. DANIEL SKODLERACK

Problem 1 (10*+10* points, Cramer's rule)**.** Consider the following matrix A and vector b.

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}, \; b = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

(i) Compute the solution of $Ax = b$ using Cramer's rule.
(ii) Compute the adjoint matrix of A.

Problem 2 (20 points, vector space axioms)**.** Prove Proposition 106, except of (ass +) and (ldist).

Problem 3 (6+7+7, vectors and polygons)**.** Consider the following polygon

$$Y = (0,0) + [0,1](1,1) + [0,1](2,1) + [0,1](1,2).$$

(i) Draw Y.
(ii) Describe Y using inequalities.
(iii) Compute the diameter of Y, i.e.

$$\sup\{d(P,Q) |\; P, Q \in Y\}.$$

At which points the diameter is realised?

Problem 4 (15 points, polytope)**.** Consider the n-space with point space $X = \mathbb{R}^n$ and vector space $V = \mathbb{R}^n$.

(i) Let P be a point of X and L be an affine line contained in X which does not contain P. Let Q be a point on L. Prove that for every $\delta > 0$ there is a point $Q_\delta \neq Q$ on L such that $d(P, Q_\delta) > d(P, Q)$ and $d(Q, Q_\delta) < \delta$.
(ii) ($n = 3$) Consider the following subset:

$$Y := \{(1,1,1) + \lambda_1 \overrightarrow{(1,2,2)} + \lambda_2 \overrightarrow{(-1,2,1)} \;|\; \lambda_1, \lambda_2 \in [0,1]\}$$

Find in Y all pairs of points P_0, Q_0 of maximal distance to each other, i.e. such that

$$d(P_0, Q_0) = \sup\{d(P,Q) \;|\; P, Q \in Y\}$$

. (You have to prove your result. Hint: (i) could help.)

Problem 5 (20 points, angles of a polygon))**.** Consider the polygon with the vertexes

$$A = (1,2),\; B = (2,3),\; C = (\frac{5}{2}, \frac{6-\sqrt{3}}{2}),\; D = (\frac{3}{2}, \frac{6-\sqrt{3}}{2})$$

in that order. Compute the angles at the vertexes,

$$\angle(D,A,B),\; \angle(A,B,C),\; \angle(B,C,D),\; \angle(C,D,A),$$

Date: Please hand in before the lecture by **30th of October 2024**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference. * questions give extra points.