Exercise Set 1.9

1. The accompanying figure shows a network in which the flow rate and direction of flow in certain branches are known. Find the flow rates and directions of flow in the remaining branches.

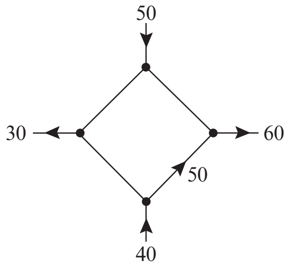


Figure Ex-1

2. The accompanying figure shows known flow rates of hydrocarbons into and out of a network of pipes at an oil refinery.

(a) Set up a linear system whose solution provides the unknown flow rates.

(b) Solve the system for the unknown flow rates.

(c) Find the flow rates and directions of flow if $x_4 = 50$ and $x_6 = 0$.

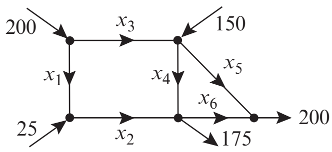


Figure Ex-2

3. The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour.

(a) Set up a linear system whose solution provides the unknown flow rates.

(b) Solve the system for the unknown flow rates.

(c) If the flow along the road from A to B must be reduced for construction, what is the minimum flow that is required to keep traffic flowing on all roads?

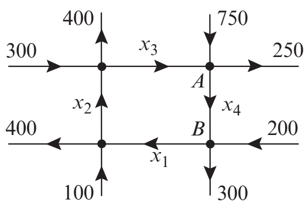


Figure Ex-3

4. The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour.

(a) Set up a linear system whose solution provides the unknown flow rates.

(b) Solve the system for the unknown flow rates.

(c) Is it possible to close the road from A to B for construction and keep traffic flowing on the other streets? Explain.

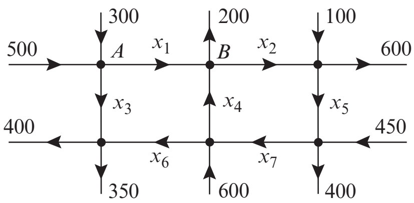
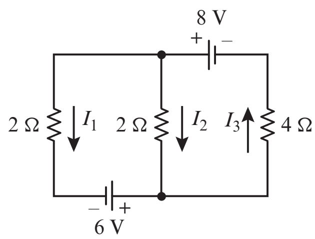


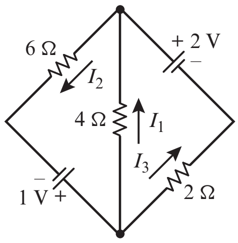
Figure Ex-4

In Exercises 5–8, analyze the given electrical circuits by finding the unknown currents.

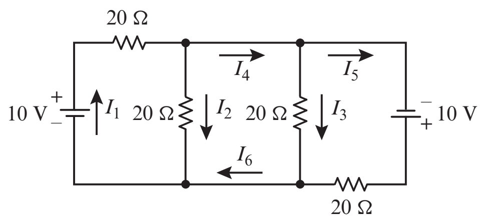
5.



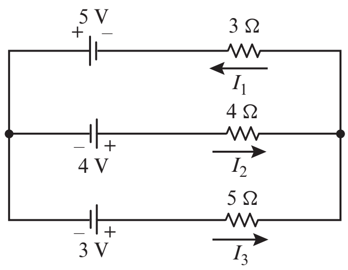
6.



7.



8.



8. Consider an open economy with consumption matrix

$$C = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{8} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{8} \end{bmatrix}$$

If the open sector demands the same dollar value from each product-producing sector, which such sector must produce the greatest dollar value to meet the demand? Is the economy productive?

9. Consider an open economy with consumption matrix

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & 0 \end{bmatrix}$$

Show that the Leontief equation $\mathbf{x} - C\mathbf{x} = \mathbf{d}$ has a unique solution for every demand vector $\mathbf{d}$ if $c_{21}c_{12} < 1 - c_{11}$.

Working with Proofs

10. (a) Consider an open economy with a consumption matrix C whose column sums are less than 1, and let $\mathbf{x}$ be the production vector that satisfies an outside demand $\mathbf{d}$; that is, $(I - C)^{-1}\mathbf{d} = \mathbf{x}$. Let $\mathbf{d}_j$ be the demand vector that is obtained by increasing the jth entry of $\mathbf{d}$ by 1 and leaving the other entries fixed. Prove that the production vector $\mathbf{x}_j$ that meets this demand is

$$\mathbf{x}_j = \mathbf{x} + j\text{th column vector of } (I - C)^{-1}$$

(b) In words, what is the economic significance of the jth column vector of $(I - C)^{-1}$? [*Hint:* Look at $\mathbf{x}_j - \mathbf{x}$.]

11. Prove: If C is an $n \times n$ matrix whose entries are nonnegative and whose row sums are less than 1, then $I - C$ is invertible and has nonnegative entries. [*Hint:* $(A^T)^{-1} = (A^{-1})^T$ for any invertible matrix A.]

True-False Exercises

TF. In parts (a)–(e) determine whether the statement is true or false, and justify your answer.

(a) Sectors of an economy that produce outputs are called open sectors.

(b) A closed economy is an economy that has no open sectors.

(c) The rows of a consumption matrix represent the outputs in a sector of an economy.

(d) If the column sums of the consumption matrix are all less than 1, then the Leontief matrix is invertible.

(e) The Leontief equation relates the production vector for an economy to the outside demand vector.

Working with Technology

T1. The following table describes an open economy with three sectors in which the table entries are the dollar inputs required to produce one dollar of output. The outside demand during a 1-week period if \$50,000 of coal, \$75,000 of electricity, and \$1,250,000 of manufacturing. Determine whether the economy can meet the demand.

Input Required per Dollar Output

Provider	Electricity	Coal	Manufacturing
Electricity	\$ 0.1	\$ 0.25	\$ 0.2
Coal	\$ 0.3	\$ 0.4	\$ 0.5
Manufacturing	\$ 0.1	\$ 0.15	\$ 0.1

Chapter 1 Supplementary Exercises

▶ In Exercises 1–4 the given matrix represents an augmented matrix for a linear system. Write the corresponding set of linear equations for the system, and use Gaussian elimination to solve the linear system. Introduce free parameters as necessary. ◀

1. $\begin{bmatrix} 3 & -1 & 0 & 4 & 1 \\ 2 & 0 & 3 & 3 & -1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 4 & -1 \\ -2 & -8 & 2 \\ 3 & 12 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -4 & 1 & 6 \\ -4 & 0 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{bmatrix}$

4. $\begin{bmatrix} 3 & 1 & -2 \\ -9 & -3 & 6 \\ 6 & 2 & 1 \end{bmatrix}$

5. Use Gauss–Jordan elimination to solve for x' and y' in terms of x and y.

$$x = \tfrac{3}{5}x' - \tfrac{4}{5}y'$$
$$y = \tfrac{4}{5}x' + \tfrac{3}{5}y'$$

6. Use Gauss–Jordan elimination to solve for x' and y' in terms of x and y.

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

7. Find positive integers that satisfy

$$x + y + z = 9$$
$$x + 5y + 10z = 44$$