LINEAR ALGEBRA 1 (FALL 2024)
PROBLEM SHEET 3

PROF. DANIEL SKODLERACK

Problem 1 (10* points, Theorem 25, uniqueness of r.r.e.f.)**.** Read the proof of Theorem 25 published on the webpage and find the typos in the proof.

Problem 2 (10+10 points, invertibility for matrices)**.** Which of the following matrices is invertible and if so compute the inverse. For the non-invertible matrix, solve the homogenous linear system. Which are the pivotal variables and which are the free variables.

(i)

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & 3 & 2 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & -4 & -2 & 0 \end{pmatrix}.$$

(ii)

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 3 & -1 \\ 1 & 3 & 1 & 2 \\ 1 & 0 & 2 & 1 \end{pmatrix}.$$

Problem 3 (10+10* points, invertible matrices)**.**

(i) Suppose we are given three matrices $A \in \mathbb{R}^{m\times m}$, $B \in \mathbb{R}^{m\times l}$ and $C \in \mathbb{R}^{l\times l}$ (m, l are positive integers). Prove that $\begin{pmatrix} A & B \\ & C \end{pmatrix}$ is invertible if and only if A and C are invertible.

(ii) A matrix $A \in \mathbb{R}^{m\times m}$ is called upper triangular if the entries satisfy $a_{i,j} = 0$ for all $i > j$. Prove that an upper triangular matrix A is invertible if and only if all diagonal entries are non-zero.

Problem 4 (10+10* points, Leontief model)**.** Solve Problem 8 on Page 101, see Supplementary exercises for Chapter 1 in the textbook. The check of the productiveness is the bonus part.

Problem 5 (10+10* points, electric circuit)**.** Solve the following exercises from the textbook, see Page 94.

(i) Exercise set 1.9 Problem 6
(ii) Exercise set 1.9 Problem 8

Date: Please hand in before the lecture by **16th of October 2024**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference. * questions give extra points.