

LINEAR ALGEBRA 1 FALL 2024
PROBLEM SHEET 1

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Problem 1 (10 points, Morgan's law). Prove the second Morgan's law, see Proposition 9(b).

Problem 2 (10 points, truth table). Compute the truth table for the following logical expression.

$$((r \vee (p \vee \neg q)) \Rightarrow (r \wedge (q \vee p))) \Rightarrow ((r \vee p) \wedge (r \vee q))$$

Problem 3 (5+5 points, row echelon form). Let a be a real number.

(i) Find a row echelon form for the matrix

$$(0.1) \quad \begin{pmatrix} 1 & 1 & 2 & 2 & 1 \\ -1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 3 & 0 & 3 \\ 2 & 1 & 1 & 3 & a \end{pmatrix}.$$

Which are the pivot columns and the pivot positions?

(ii) Can you find the reduced row echelon form for (0.1)?

Problem 4 (10 points, linear system). Is there a linear system (with real variables) such that its set of solutions is the set of integers? More precisely: Are there positive integers m, n and a linear system with m equations and n variables such that its set of solutions is

$$\{(z, 0, 0, \dots, 0) \in \mathbb{R}^n \mid z \in \mathbb{Z}\}.$$

Give a proof for your answer.

Date: Please hand in before the lecture by **25thth of September 2024**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.