# LINEAR ALGEBRA 1
# PROBLEM SHEET 8

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**Problem 1** (10, Span as a minimal subspace)**.** Let W be the intersection of all subspacs of $\mathbb{R}^4$ which contain the vectors $(1, 0, 1, 1), (1, 1, 1, 1)$ and $(1, 2, 1, 1)$. Find a basis for W.

**Problem 2** (20, Fundamental spaces of a matrix)**.** Consider the following matrix A.

$$\begin{pmatrix} -1 & 2 & -7 & 1 & 1 \\ 1 & 0 & 5 & 3 & 6 \\ 2 & 1 & 6 & 5 & 1 \\ 1 & 1 & -2 & -1 & -2 \\ 1 & -1 & 7 & 2 & 1 \end{pmatrix}$$

Compute:

(i) a basis for the column space of A,
(ii) a basis for the row space of A,
(iii) the null space of A,
(iv) the rank and the nullity of A.

**Problem 3** (20, base extension theorem)**.**

(i) Prove the base extension theorem, see Theorem 183.
(ii) Prove Proposition 190 in the notes: Let V be a finite dimensional vector space and $W_1$, $W_2$ be subspaces of V. Show that

$$\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2).$$

**Problem 4** (10+10+10, infinite dimension)**.** We also have the notion of linear independence for infinite sets: Let V be a vector space and T be a subset of V. We call T *linearly independent* if all finite subsets of T are linearly independent, see Definition 168 for the finite sets. A subset B of V is called a *basis* of V if it spans V and is linearly independent.

(i) Prove: V is finitely generated, i.e. there exists a finite set S which spans V, if and only if every linearly independent subset of V is finite.
(ii) Show that Poly($\mathbb{R}$) (the set of polynomial functions of $\mathbb{R}$) is not finitely generated.
(iii) Find a basis for

$$\{p \in \text{Poly}(\mathbb{R}) \mid \ \forall x \in \mathbb{R} : \ p(-x) = -p(x)\},$$

the space of odd polynomial functions.

*Date*: Please hand in before the lecture by **29th of November 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference. The intermediate steps for computations need to be provided.