LINEAR ALGEBRA 1
PROBLEM SHEET 6

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Problem 1 (20 points, angles of a polygon)**.** Consider the polygon with the vertexes

$$A = (1,2),\ B = (2,3),\ C = (\frac{5}{2}, \frac{6-\sqrt{3}}{2}),\ D = (\frac{3}{2}, \frac{6-\sqrt{3}}{2})$$

in that order. Compute the angles at the vertexes,

$$\angle(D,A,B),\ \angle(A,B,C),\ \angle(B,C,D),\ \angle(C,D,A),$$

Problem 2 (10+10*+10 points, generalized parallelogram law)**.**

(i) (Parallelepiped law) Prove for $v, w, u \in \mathbb{R}^n$

$$\|u+v+w\|^2 + \|u-v+w\|^2 + \|u+v-w\|^2 + \|u-v-w\|^2 = 4(\|u\|^2 + \|v\|^2 + \|w\|^2)$$

(ii) (*) Generalize and prove (i) for a paralleletope spanned by m vectors, $m \geqslant 2$.

(iii) Let A, B, C, D be points in $\mathbb{R}^n$ which satisfy

$$\|\overrightarrow{AC}\|^2 + \|\overrightarrow{BD}\|^2 = \|\overrightarrow{AB}\|^2 + \|\overrightarrow{BC}\|^2 + \|\overrightarrow{CD}\|^2 + \|\overrightarrow{DA}\|^2$$

Prove that $\overrightarrow{AB} = \overrightarrow{DC}$. (Hint: Draw a picture.)

Problem 3 (20 points, distance of a point to an affine subspace)**.** Find the distance of the given point P_0 to the given set S.

(i) $P_0 = (1,-1)$, S is the line through $(2,3)$ and $(1,0)$.

(ii) $P_0 = (1,0,1)$, S is the plane through $(1,1,2), (0,1,-1)$ and $(0,0,2)$.

(iii) $P_0 = (1,0,0,0)$, S is the line through $(5,1,2,1)$ and $(4,1,1,1)$.

(iv) $P_0 = (1,0,0,0)$, S is the plane through $(-1,1,2,3)$, $(0,1,1,0)$ and $(1,1,-1,2)$.

Problem 4 (20 points, lines and planes)**.** Describe the following sets with (a) a vector form, (b) parametric equations and (c) as an intersection of hyperplanes.

(i) The line through the points $(1,0,0,1)$ and $(2,1,0,1)$.

(ii) The plane throught the points $(1,1,1,-1), (1,0,1,0)$ and $(1,2,1,2)$.

Date: Please hand in before the lecture by **15th of November 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference. The intermediate steps for computations need to be provided.