LINEAR ALGEBRA 1
PROBLEM SHEET 3

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Problem 1 (5+5 points, unique solutions)**.**
(i) Read the proof of Theorem 46 and complete the proof.
(ii) Let $A \in \mathbb{R}^{m\times m}$ and $b \in \mathbb{R}^{m\times 1}$ be given. Prove that the following assertions are equivalent.
  (a) A is invertible.
  (b) The equation $Ax = b$, $x \in \mathbb{R}^{m\times 1}$ has exactly one solution.

Problem 2 (15+5 points, invertibility for matrices)**.** Which of the following matrices is invertible and if so compute the inverse.

(i)
$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 3 & 0 & 3 \end{pmatrix}.$$

(ii)
$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 \end{pmatrix}.$$

For the non-invertible matrix, solve the homogenous linear system. Which are the pivotal variables and which are the free variables.

Problem 3 (10 points, invertible matrices)**.** Let A, B, C, D be $m \times m$ matrices such that the product $ABCD$ is invertible. Prove that A, B, C, D are invertible matrices..

Problem 4 (10 points, matrix with huge diagonal entries)**.** Decide wether or not the following matrix is invertible.
$$\begin{pmatrix} 1000 & 2 & 1 & 3 \\ 1 & 800 & 1 & 2 \\ 1 & 1 & 700 & 1 \\ 1 & 1 & 0 & 5 \end{pmatrix}$$

Problem 5 (20* points, uniqueness of reduced echelon form)**.** Prove Theorem 25.

Date: Please hand in before the lecture by **25th of October 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference. * questions give extra points.