LINEAR ALGEBRA 1
PROBLEM SHEET 2

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Problem 1 (10, elementary row operations). Consider the row operations (E4)="Adding a row to another row" and (E1),(E2),(E3) from Definition 20. Show that the pair of row operations (E1),(E4) is equivalent to the triple (E1),(E2),(E3), i.e. we can obtain (E2) and (E3) using a sequence of row operations using (E1),(E4).

Problem 2 (20, circle equation). Find the mid point and the radius of the circle containing the points $(1,0), (0,2), (2,2)$. Find all $a, b, c, d, e \in \mathbb{R}$ such that the above points all solve

$$ax^2 + by^2 + cx + dy + e = 0$$

What conditions do we need to obtain an equation describing a circle? What is the geometric interpretation in the other cases for those coefficients.

Problem 3 (10, points hyperplanes). Let H_1 and H_2 be hyperplanes in $\mathbb{R}^n$ such that $H_1 \subseteq H_2$. Prove that $H_1 = H_2$.

Problem 4 (10 points, pivot positions). Find the pivot positions and pivot culomns of the matrix

$$\begin{pmatrix} 2 & 1 & 1 & 5 & 1 \\ 3 & 5 & -1 & 2 & 2 \\ -3 & 2 & -2 & 3 & 5 \\ 3 & 2 & 1 & 4 & 3 \\ 5 & 10 & -1 & 14 & 11 \end{pmatrix}.$$