DIFFERENTIAL TOPOLOGY
PROBLEM SHEET 13

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Problem 1 (10, proper maps)**.** Let $f : \mathbb{R} \to \mathbb{R}$ be a polynomial function. Give a sufficient and necessary condition on the coefficients of f for f to be a proper map. In the case of properness compute the degree of f as a smooth map.

Problem 2 (10, equivalent characterization of properness)**.** Let $f : \mathrm{M} \to \mathrm{N}$ be a continuous map between smooth manifolds. Then are equivalent:

(i) f is proper.

(ii) f is closed, i.e. sends closed sets to closed sets, and for every point $y \in \mathrm{N}$ the fiber $f^{-1}(y)$ is compact.

(iii) For all smooth manifolds Z the map

$$f_{\mathrm{Z}} : \mathrm{M} \times \mathrm{Z} \to \mathrm{N} \times \mathrm{Z}, \;\; f(x, z) := (f(x), z)$$

is a closed map.

Problem 3 (10, bump forms)**.** Give a proof for Lemma 5.7 for non-orientable manifolds.

Problem 4 (10, closed orientable 4-manifolds)**.** For which non-negative integers n there is connected, closed, orientable 4-manifold with first de Rham cohomology of dimension n?

Date: Please hand in before the lecture by **19th of May 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.