DIFFERENTIAL TOPOLOGY
PROBLEM SHEET 12

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Problem 1 (10, Mayer-Vietoris sequence)**.** Prove Proposition 5.14. about the Mayer-Vietoris sequence for the de Rham cohomology with compact support.

Problem 2 (10, non-degenerate bilinear forms)**.** Let V and W be finite diensional real vector spaces together with a real bilinear form $b : \mathrm{W} \times \mathrm{V} \to \mathbb{R}$. Show the equivalence of the following assertions.

(i) b is non-degenerate.
(ii) The canonical map $\mathrm{V} \to \mathrm{W}^*$ is an $\mathbb{R}$-linear isomorphism.
(iii) The canonical map $\mathrm{W} \to \mathrm{V}^*$ is an $\mathbb{R}$-linear isomorphism.

Problem 3 (10, Poincaré duality for the base case)**.** Show the Poincaré duality for $\mathbb{R}^n$, i.e. the non-degeneracy of the integral form.

Problem 4 (10, integral over a manifold)**.** Compute the integral

$$\int_{\mathrm{M}} (x + e^y) dy$$

for M being the manifold given by the set of all solutions of

$$16x^2 + 9y^2 = 144, x, y \in \mathbb{R},$$

with clockwise orientation.

Date: Please hand in before the lecture by **12th of May 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.