DIFFERENTIAL TOPOLOGY
PROBLEM SHEET 11

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Problem 1 (10 points, vector fields on S^2)**.** Let X be a global smooth vector field on S^2. Prove that X has a zero.

Problem 2 (10 points, torus)**.** Compute $\int_{S^1\times S^1}\omega$ with respect to the outwards pointing orientation and where ω is the 2-form with integral

$$\int_{S^1\times S^1}\cos^2(\theta_2)\omega = 1.$$

Problem 3 (10 points, orientation)**.** Finish the proof of Proposition 4.5. It is the Proposition about the characterization of orientation.

Problem 4 (10 points, cylinder)**.** Let M be the lateral part of a cylinder

$$M = \{(x,y,z) \mid \ x^2+y^2 = R^2,\ -1 \leqslant z \leqslant 1\},\ (R>0 \text{ fixed})).$$

Compute $\int_M z^2 dz \wedge d\theta$. where the orientation is given by the outwards pointing normal vector, with respect to the full cylinder.

Date: Please hand in before the lecture by **28th of April 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.