

DIFFERENTIAL TOPOLOGY
PROBLEM SHEET 10

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Problem 1 (20 points, Mayer-Vietoris sequence). Complete the proof of Theorem 3.69, i.e. show that the Mayer-Vietoris sequence is exact and the connecting morphism δ is well-defined.

Problem 2 (10 points, finite dimensional de Rham cohomology). Let M be a closed manifold (by definition: compact without boundary). Show that $H_{dR}^*(M)$ is finite dimensional.

Problem 3 (10 points, sphere). Compute the de Rham cohomology of the n -dimensional sphere.

Problem 4 (10 points, torus). Compute the de Rham cohomology of the Kleinian bottle.

Date: Please hand in before the lecture by **21th of April 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.