DIFFERENTIAL TOPOLOGY
PROBLEM SHEET 6

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Problem 1 (10 points, open sets)**.** Let M, N be C^r-manifolds and V be a non-empty open subset of N. Then $\mathrm{C}^r(\mathrm{M},\mathrm{V})$ is an open subset of $\mathrm{C}^r_S(\mathrm{M},\mathrm{N})$.

Problem 2 (20 points, neighborhood basis)**.**

(i) Let X be a topological space and $(\mathrm{A}_i)_{i\in\mathrm{I}}$ be a locally finite family of closed subsets of X. Show that the union of all A_i is a closed subset of X.

(ii) Let M be a paracompact Hausdorff space and let $(\mathrm{K}_i)_{i\in\mathrm{I}}$ be a locally finite family of compact subsets of M. Show that there is a locally finite open covering $\mathfrak{U} = (\mathrm{U}_j)_{j\in\mathrm{J}}$ of M such that for every index $i \in \mathrm{I}$ there is a $j \in \mathrm{J}$ such that K_i is a subset of U_j.

(iii) Let r be a non-negative integer and M be a C^r-manifold and let f be a C^r-map from M to $\mathbb{R}$. Prove that the following sets form an open neighborhood basis of f in the strong topology:

$$\mathcal{N}(f, \Phi, K, E),$$

where

(a) $\mathrm{K} = (\mathrm{K}_i)_{i\in\mathrm{I}}$ is a locally finite compact covering of M and $\Phi = ((\phi_i, \mathrm{U}_i))_{i\in\mathrm{I}}$ is a family of charts such that K_i is contained in U_i,

(b) $E = (\epsilon_i)_{i\in\mathrm{I}}$ is a family of positive numbers and

(c) $\mathcal{N}$ is defined to be the set of all C^r-maps h from M to $\mathbb{R}$ such that for all indexes $i \in \mathrm{I}$ we have that $\|h - f\|_{r,\mathrm{K}_i}$ is smaller than ϵ_i.

Problem 3 (10 points, differential structures without common chart)**.** Consider the standard C^i-structure α_i on $\mathbb{R}$, i.e. the one given by the identity map as a global chart, for $i = 1, 2$. Is there another C^2-structure β_2 on $\mathbb{R}$ which does not intersect α_2 but is contained in α_1?

Problem 4 (10 points, C^∞-differential structures on $\mathbb{R}$)**.** How many C^∞-differential structures does $\mathbb{R}$ with Euclidean topology has up to C^∞-diffeomorphism? Give a proof. *Hint: You just need to consider C^1-differential structures.*

Date: Please hand in before the lecture by **24.03.2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.