DIFFERENTIAL TOPOLOGY
PROBLEM SHEET 5

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Problem 1 (10, shrinking)**.** Let M be a smooth manifold and $\mathfrak{U} = (\mathrm{U}_i)_{i\in\mathrm{I}}$ be a locally finite open covering using charts, such that U_i has a compact closure in M. Show that there is an open cover of M which is a shrinking of $\mathfrak{U}$.

Problem 2 (10, submersions)**.** Let $r \geqslant 1$. Show that $\mathrm{Subm}^r(\mathrm{M}, \mathrm{N})$ is open in $\mathrm{C}^r(\mathrm{M}, \mathrm{N})$.

Problem 3 (20, strong topology)**.**

(i) Show that for $0 \leqslant r < s$ the inclusion map

$$\mathrm{C}^s_S(\mathrm{M}, \mathrm{N}) \to \mathrm{C}^r_S(\mathrm{M}, \mathrm{N})$$

is continuous.

(ii) Let M, N be smooth manifolds and M be compact. Show that the strong and the weak topology on $\mathrm{C}^r(\mathrm{M}, \mathrm{N})$ coincide.

Problem 4 (10, partition of unity)**.** Give a solution for Example 2.7(b).

Date: Please hand in before the lecture by **March 17th 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.