DIFFERENTIAL TOPOLOGY
PROBLEM SHEET 4

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Problem 1 (10 points, immersions with dense image)**.** Show that there is a smooth immersion from $\mathbb{R}$ to $\mathbb{R}^2$ with dense image.

Problem 2 (10 points, Prop 1.34.)**.** Prove Proposition 1.34 for the case of general manifolds, i.e. including manifolds with boundary.

Problem 3 (10 points, submanifolds)**.** We consider Definition 1.6 of a submanifold for manifolds without boundary. Let $r \geqslant 1$ and M_2 be a C^r-submanifold of M_3 and M_1 be subset of M_2. Then are equivalent:

(i) M_1 is a C^r-submanifold of M_3.
(ii) M_1 is a C^r-submanifold of M_2.

Problem 4 (10 points, submanifolds)**.** Let $r \geqslant 0$. Show that for manifolds without boundary the two definitions of submanifold, Definition 1.6 and Definition 1.57, are equivalent.

Date: Please hand in before the lecture by **March 10th 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.