

DIFFERENTIAL TOPOLOGY
PROBLEM SHEET 3

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Problem 1 (10, a ray embedded in a non-compact manifold). Let M be a smooth manifold. Show that the following statements are equivalent.

- (i) 1° The manifold M is not compact.
- (ii) 2° There is a closed C^1 -embedding of $[0, \infty[$ into M . (“closed” here means that the image is closed in M .)

Problem 2 (20, non-trivial tangent bundle). Prove that the tangent bundle of the real projective plane is not trivial.

Problem 3 (10, submersions). Prove that a submersion is an open map, i.e. maps open sets onto open sets.

Problem 4 (10, trivialization). Prove Proposition 1.31.

Date: Please hand in before the lecture by **March 3rd 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.