DIFFERENTIAL TOPOLOGY
PROBLEM SHEET 1

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Problem 1 (10, maximal atlas)**.** Let M be a C^r-manifold and Φ is a C^r-atlas on M. Show that there is a unique maximal C^r-atlas on M which contains Φ.

Problem 2 (20, manifolds as fibers)**.**

(i) Let $f : \mathbb{R}^{n+1} \to \mathbb{R}$ be a C^1-map and $y \in \mathrm{im}(f)$ such that at every element in the pre-image M of y the derivative of f is non-zero. Show that M is a C^1-submanifold of $\mathbb{R}^{n+1}$.

(ii) Is the above statement still true if we allow M to contain one element where the derivative of f is zero?

Problem 3 (10, atlas on the n-dimensional sphere)**.** We consider the C^1-differential structure α_1 on S^n ($n \geqslant 1$) containing the atlas of Example 1.5(a). Find in α_1 an atlas of minimal cardinality.

Problem 4 (10, graph of a function)**.** Let $f : \mathbb{R}^n \to \mathbb{R}^k$ be a continuous map and Γ_f its graph. Then Γ_f has a C^ω-differential structure C^ω-diffeomorphic to $\mathbb{R}^n$. Prove that the following assertions are equivalent:

(i) The inclusion $\Gamma_f \hookrightarrow \mathbb{R}^{n+k}$ is C^1-differentiable.
(ii) $f \in C^1(\mathbb{R}^n, \mathbb{R}^k)$.
(iii) Γ_f is a C^1-submanifold of $\mathbb{R}^{n+k}$.

Date: Please hand in before the lecture by **17th of February 2023**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.