INTRODUCTION TO MATHEMATICAL FINANCE
PROBLEM SHEET 11

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Problem 1 (10, Cox-Ross-Rubinstein model)**.** Show that the CRR model is arbitrage-free if the parameters satisfy $a < r < b$.

Problem 2 (10, Δ-hedge)**.** Consider the CRR model with the parameters

$$(a, r, b, \mathrm{T}, \mathrm{S}_0) = (0.1, 0.2, 0.3, 3, 1).$$

Compute the price and the Δ-hedge for the straddle on S. (Reminder: A straddle on S is the option $\mathrm{C} = |\mathrm{S} - \mathrm{S}_0|$.)

Problem 3 (10, Δ-hedge proposition)**.** Prove Proposition 111.

Problem 4 (10, weak Markov property)**.** Let $(\Omega, \mathcal{F}, \mathrm{P})$ be a probability space with filtration $(\mathcal{F}_t)_{t=\overline{0,\mathrm{T}}}$. Let $(\mathrm{X}_t)_{t=\overline{0,\mathrm{T}}}$ be a one-dimensional adapted process which satisfies that for every pair $0 \leqslant s \leqslant t \leqslant T$ the increment $\mathrm{X}_t - \mathrm{X}_s$ is independent to $\mathcal{F}_s$. Show that X is a weak Markov process, i.e. X satisfies the weak Markov property: For every bounded Borel measurable map $f : \mathbb{R} \to \mathbb{R}$ the conditional expactitions satisfy

$$\mathrm{E}[f(\mathrm{X})|\mathcal{F}_s] = \mathrm{E}[f(\mathrm{X})|\sigma(\mathrm{X}_s)], 0 \leqslant s < t \leqslant \mathrm{T}.$$

(That means that the expected performance for $t > s$ of a derivative of X does only depend on X_s and not on the performance of X before s.)

Date: Please hand in before the lecture by **24th. of May 2022**. For all exercises the results need to be proven using results from this lecture and the lectures before, provided you give a reference.