COMMUTATIVE ALGEBRA
EXERCISE SHEET 13

PROF. DANIEL SKODLERACK

Problem 1 (10 points)**.** Show that a closed subset V of the Grothendieck spectrum of a ring R is irreducible if and only if I(V) is a prime ideal.

Problem 2 (10 points)**.** Let κ be a field. Show that the completion of $\mathrm{R} = \kappa[X_1, \ldots, X_n]$ with respect to the ideal $(X_1, \ldots, X_n)_{\mathrm{R}}$ is κ-algebra-homeomorphic to $\kappa[[X_1, \ldots, X_n]]$.

Problem 3 (10 points)**.** Prove Example 169(c).

Problem 4. Show that the cubes in $\mathbb{Z}_3$, i.e. the elements of the form $x^3,\ x \in \mathbb{Z}_3$, are exactly the elements zero and the ones of the form $3^{3l}a$ with $l \in \mathbb{N}_0$ and $a \in \mathbb{Z}_3$, $a \equiv_9 +1, -1$.

Date: Please hand in before the lecture on Thursday **23.12.2021**. For all exercises the results need to be proven. You are allowed to use results from the Abstract Algebra course.