## COMMUTATIVE ALGEBRA EXERCISE SHEET 11

## PROF. DANIEL SKODLERACK

**Problem 1** ((Krull's intersection theorem) 10 points). Let R be a finitely generated algebra over a field. Show that its Jacobson radical is equal to the nil radical.

**Problem 2** (10 points). Proof Proposition 143.

**Problem 3** (10 points). Prove Corollary 142.

**Problem 4** (10 points). Let  $P(X_1, ..., X_l)$  and  $Q(X_1, ..., X_l)$  be two polynomials over an algebraically closed field k. Suppose that all roots (in  $k^l$ ) of P are roots of Q. Show that all irreducible divisors of P divide Q.

Date: Please hand in before the lecture on Thursday by **02.12.2021**. For all exercises the results need to be proven. You are allowed to use results from the Abstract Algebra course.