COMMUTATIVE ALGEBRA
EXERCISE SHEET 10

PROF. DANIEL SKODLERACK

Problem 1 ((hard question) 10 points)**.** Do we have a local global principle for injectivity for finitely presented modules?

Problem 2 (10 points)**.** Prove the local global principle for projectivity for finitely presented modyules.

Problem 3 (10 points)**.** Let R be a ring. We define for an ideal $\mathfrak{a}$ of R the following subset of Spec(R):

$$\mathrm{D}(\mathfrak{a}) := \mathrm{Spec}(\mathrm{R}) \backslash \mathrm{V}(\mathfrak{a}).$$

Show that the collection of all sets $\mathrm{D}(\mathfrak{a})$, $\mathfrak{a}$ an ideal, forms a topology on Spec(R). It is called the Zariski topology of R.

Problem 4 (10 points)**.** Prove the Meta lemma.

Date: Please hand in before the lecture on Thursday by **25.11.2021**. For all exercises the results need to be proven. You are allowed to use results from the Abstract Algebra course.